UNIT 1: NUMBER SYSTEM

Exercise: 1.1

1.Is zero a rational number? Can you write it in the form pq, where p and q areintegers and q≠0?

Solution:

We know that a number 'r' is said to be a rational number if it can be representedin the form pq, where p and q are integers and q≠0.

Now, we say that zero is a rational number if it can be represented in the aboveform. Further, we see that zero can be represented as 01,02,03, etc.

Therefore, zero is a rational number.

2.Find six rational numbers between 3 and 4.

Solution:

An infinite number of rational numbers are possible between 3 and 4.

To get these numbers the criteria is that the denominator should be the sum of the2 numbers i.e. 3+4=7

Now, 3 and 4 can be represented in fractions by multiplying and dividing them by7.

3=3 ×77=217 **and** 4=4 ×77=287

The required rational numbers can be found by changing the numerators from 21 to 28. Therefore, the rational numbers are 227,237,247,257,267,277.

3.Find five rational numbers between 35 **and** 45.

Solution:

There are infinite number of rational numbers between 35 **and** 45.

We can find them by multiplying and dividing the numerator and denominator by a number. We are doing this so that the gap between the numerators of the two numbers increase and we can easily select the required numbers.

We can choose any number to multiply and divide but ideally, we choose the number that is more than the required number (here, 5).

So, let us choose 7. Now, 35=3×75×7=2135 **and** 45=4×75×7=2835.

Therefore, the required numbers are 2235,2335,2435,2535,2635.

4. State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Solution:

(i) True, because we can say that whole numbers are nothing but natural numbers plus zero. Therefore, every natural number is a whole number but every whole number is not a natural number. As 0 is not a natural number.

(ii) False, because integers include both positive and negative numbers. The whole numbers include only positive numbers and negative numbers are not whole numbers. Therefore, every integer is a not a whole number.

(iii) False, because rational numbers can also be in the form of fractions and these fractional numbers are not whole numbers. For example, 23,35,43 are not whole numbers but they are rational numbers.

Exercise: 1.2

1. State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Solution:

(i) True, since the real numbers are nothing but a combination of rational and irrational numbers.

(ii) False, because the negative numbers on the number line cannot be expressed in the form $\sqrt{m}$.

(iii) False, because real numbers contain both rational and irrational numbers. Therefore, every real number cannot be irrational.

2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Solution:

No, the square root of all positive numbers need not be irrational

For example, $\sqrt{4}=2$ and $\sqrt{9}=3$.

Here, 2 and 3 are rational.

Therefore, the square roots of all positive integers are not irrational.

3. Show how $\sqrt{5}$ can be represented on the number line.

Solution:

To represent $\sqrt{5}$ on the number line, take $OB=2$ units and make a perpendicular at B so that $AB=1$ unit.

By Pythagoras theorem, we get $OA^2=OB^2+AB^2$.

So, $OA^2=2^2+1^2$ $OA^2=5$ $OA=\sqrt{5}$

Now, taking O as center and OB as radius, draw an arc intersecting number line at C.

OC is the required distance that represents $\sqrt{5}$.

UNIT 2: POLYNOMIALS

Exercise: 2.1

1.Which of the following expressions are polynomials in one variable and which arenot? State reasons for your answer.

(i)$4x^2-3x+7$

(ii)$y^2+\sqrt{2}$

(iii)$3\sqrt{t}+t\sqrt{2}$

(iv)$y+\frac{2}{y}$

(v)$x^{10}+y^3+t^{50}$

Solution:

(i)Given expression is a polynomial

It is of the form $a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$ where $a_n,a_{n-1},\ldots a_0$are constants. Hence given expression $4x^2-3x+7$ is a polynomial.

(ii)Given expression is a polynomial

It is of the form $a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$ where $a_n,a_{n-1},\ldots a_0$are constants. Hence given expression $y^2+\sqrt{2}$ is a polynomial.

(iii)Given expression is not a polynomial. It is not in the form of$a_nx^n+a_{n-1}2^{n-1}+\cdots+a_1x+a_0$

where $a_n,a_{n-1},\ldots a_0$ all constants.

Hence given expression $3\sqrt{t}+t\sqrt{2}$ is not a polynomial.

(iv)Given expression is not a polynomial$y+2y=y+2.y^{-1}$

It is not of form $a_nx^n+a_{n-1}x^{n-1}+\cdots+a_0$, where $a_n,a_{n-1},\ldots a_0$ are constants.

Hence given expression $y+\frac{2}{y}$ is not a polynomial.

(v)Given expression is a polynomial in three variables. It has three variablesx,y,t.

Hence the given expression $x^{10}+y^3+t^{50}$ is not a polynomial in one variable.

2. Write the coefficients of x^2 in each of the following:

(i) $2+x^2+x$

(ii) $2-x^2+x^3$

(iii) $\frac{\pi}{2}x^2+x$

(iv) $\sqrt{2}x-1$

Solution:

(i) The constant multiplied with the term x^2 is called the coefficient of the x^2.

Given polynomial is $2+x^2+x$.

Hence, the coefficient of x^2 in given polynomial is equal to 1.

(ii) The constant multiplied with the term x^2 is called the coefficient of the x^2.

Given polynomial is $2-x^2+x^3$.

Hence, the coefficient of x^2 in given polynomial is equal to -1.

(iii) The constant multiplied with the term x^2 is called the coefficient of the x^2.

Given polynomial is $\frac{\pi}{2}x^2+x$.

Hence, the coefficient of x^2 in given polynomial is equal to $\frac{\pi}{2}$.

(iv) The constant multiplied with the term x^2 is called the coefficient of the x^2.

Given polynomial is $\sqrt{2}x-1$.

In the given polynomial, there is no x^2 term.

Hence, the coefficient of x^2 in given polynomial is equal to 0.

3. Give one example each of a binomial of degree 35 and of a monomial of degree 100.

Solution:

Degree of polynomial is highest power of variable in the polynomial. And number of terms in monomial and binomial respectively equals to one and two.

A binomial of degree 35 can be $x^{35}+7$

A monomial of degree 100 can be $2x^{100}+9$

4. Write the degree of each of the following polynomials

(i) $5x^3+4x^2+7x$

(ii) $4-y^2$

(iii) $5tt-\sqrt{7}$

(iv) 3

Solution:

(i) Degree of polynomial is highest power of variable in the polynomial.

Given polynomial is $5x^3+4x^2+7x$

Hence, the degree of given polynomial is equal to 3.

(ii) Degree of polynomial is highest power of variable in the polynomial.

Given polynomial is $4-y^2$

Hence, the degree of given polynomial is 2.

(iii) Degree of polynomial is highest power of variable in the polynomial

Given polynomial is $5t-\sqrt{7}$

Hence, the degree of given polynomial is 1.

(iv) Degree of polynomial 1, **highest power of variable in the polynomial.**

Given polynomial is 3.

Hence, the degree of given polynomial is 0.

5. Classify the following as linear, quadratic and cubic polynomials.

(i) x^2+x

(ii) $x-x^3$

(iii) $y+y^2+4$

(iv) $1+x$

(v) $3t$

(vi) r^2

(vii) $7x^3$

Solution:

(i) Linear, quadratic, cubic polynomials have degrees $1,2,3$ **respectively.**

Given polynomial is x^2+x

It is a quadratic polynomial as its degree is 2.

(ii) Linear, quadratic, cubic polynomials have its degree 1,2,3 respectively.

Given polynomial is $x-x^3$.

It is a cubic polynomial as its degree is 3.

(iii) Linear, quadratic, cubic polynomial has its degree 1,2,3 respectively.

Given polynomial is $y+y^2+4$.

It is a quadratic polynomial as its degree is 2.

(v) Linear, quadratic, cubic polynomial has its degree 1,2,3 respectively.

Given polynomial is $1+x$.

It is a linear polynomial as its degree is 1.

(v) Linear, quadratic, cubic polynomial has its degree 1,2,3 respectively.

Given polynomial is $3t$

It is a linear polynomial as its degree is 1.

(vi) Linear, quadratic, cubic polynomial has its degree 1,2,3 respectively.

Given polynomial is r^2.

It is a quadratic polynomial as its degree is 2.

(vii) Linear, quadratic, cubic polynomial has its degree 1,2,3 respectively.

Given polynomial is $7x^3$.

It is a cubic polynomial as its degree is 3.

Exercise: 2.2

1. Find the value of the polynomial $5x-4x^2+3$ at

(i) $x=0$

(ii) $x=-1$

(iii) $x=2$

Solution:

(i) Given polynomial is $5x-4x^2+3$

Value of polynomial at $x=0$ is $5(0)-4(0)^2+3 =0-0+3 =3$

Therefore, value of polynomial $5x-4x^2+3$ at $x=0$ is equal to 3.

(ii) Given polynomial is $5x-4x^2+3$

Value of given polynomial at $x=-1$ is $5(-1)-4(-1)^2+3 =-5-4+3 =-6$

Therefore, value of polynomial $5x-4x^2+3$ at $x=-1$ is equal to -6.

(iii) Given polynomial is $5x-4x^2+3$

Value of given polynomial at $x=2$ is $5(2)-4(2)^2+3 =10-16+3 =-3$

Therefore, value of polynomial $5x-4x^2+3$ at $x=2$ is equal to -3

2. Find $P(0)$,$P(1)$ and $P(2)$ for each of the following polynomials.

(i) $P(y)=y^2-y+1$

(ii) $P(t)=2+t+2t^2-t^3$

(iii) $P(x)=x^3$

(iv) $P(x)=(x-1)(x+1)$

Solution:

(i) Given polynomial is $P(y)=y^2-y+1$ $P(0)=(0)^2-0+1 =1$ $P(1)=(1)^2-1+1 =1$ $P(2)=(2)^2-2+1 =4-2+1 =3$

(ii) Given polynomial is $P(t)=2+t+2t^2-t^3$ $P(0)=2+0+2.(0)^2-(0)^3 =2$ $P(1)=2+1+2(1)^2-(1)^3 =4$

$P(2)=2+2+2.(2)^2-(2)^3 =4$

(iii) Given polynomial is $P(x)=x^3$ $P(0)=(0)^3=0$ $P(1)=(1)^3=1$ $P(2)=(2)^3=8$

(iv) Given polynomial is $p(x)=(x-1)(x+1)$ $P(0)=(0-1)(0+1) =(-1)(1) =-1$ $P(1)=(1-1)(1+1) =(0)(2) =0$ $P(2)=(2-1)(2+1) =3$

4. Find the zero of the polynomials in each of the following cases.

(i) $P(x)=x+5$

(ii) $P(x)=x-5$

(iii) $P(x)=2x+5$

(iv) $P(x)=3x-2$

(v) $P(x)=3x$

(vi) $P(x)=ax, a\neq0$

(vii) $P(x)=cx+d, c\neq0, c, d$ **are real numbers.**

Solution:

For a polynomial $P(x)$**, if** $x=a$ **is said to be a zero of the polynomial** $p(x)$**, then** $P(a)$ **must be equal to zero.**

(i) Given polynomial is $P(x)=x+5$

Now, $P(x)=0 \Rightarrow x+5=0 \Rightarrow x=-5$

Hence $x=-5$ **is zero of polynomial** $P(x)=x+5$

(ii) Given polynomial is $P(x)=x-5$

Now, $P(x)=0 \Rightarrow x-5=0 \Rightarrow x=5$

Hence $x=5$ **is zero of polynomial** $P(x)=x-5$**.**

(iii) Given polynomial is $P(x)=2x+5$

Now, $P(x)=0 \Rightarrow 2x+5=0 \Rightarrow x=-52$

Hence $x=-52$ **is zero of polynomial** $P(n)=2x+5$**.**

(iv) Given polynomial is $P(x)=3x-2$

Now, $P(x)=0 \Rightarrow 3x-2=0 \Rightarrow x=23$

Hence $x=23$ **is zero of polynomial** $P(n)=3x-2$

(v) Given polynomial is $P(x)=3x$

Now, $P(n)=0 \Rightarrow 3x=0 \Rightarrow x=0$

Hence $x=0$ **is zero of polynomial** $P(n)=0$**.**

(vi) Given polynomial is $P(x)=ax$

Now, $P(x)=0 \Rightarrow ax=0$

$\Rightarrow a=0$ **or** $x=0$

But given that $a\neq0$

Hence $x=0$ **is zero of polynomial** $P(x)=ax$**.**

(vii) Given polynomial is $P(x)=cx+d$ $P(x)=0 \Rightarrow cx+d=0 \Rightarrow cx=-d \Rightarrow x=-\frac{d}{c}$

Hence $x=-\frac{d}{c}$ **is zero of given polynomial** $P(n)=cx+d$

Exercise: 2.3

1. Find the remainder when x^3+3x^2+3x+1 **is divided by**

(i) $x+1$

(ii) $x-\frac{1}{2}$

(iii) x

(iv) $x+\pi$

(v) $5+2x$

Solution:

We know, the remainder of polynomial $P(n)$ **when divided by another polynomial** $(an+b)$ **where** a **and** b **are real numbers** $a\neq 0$ **is equal to** $P\left(-\frac{b}{a}\right)$**.**

(i) Given polynomial is $P(x)=x^3+3x^2+3x+1$

When $P(x)$ **is divided by** $x+1$**, then the remainder is** $P(-1)$

Hence, remainder = $P(-1)=(-1)^3+3.(-1)^2+3(-1)+1 =-1+3-3+1 =0$

Remainder when polynomial x^3+3x^2+3x+1 **is divided by** $x+1$ **is equal to** 0

(ii) Given polynomial is $P(x)=x^3+3x^2+3x+1$

When $P(x)$ **is divided by** $x-\frac{1}{2}$**, then the remainder is** $P\left(\frac{1}{2}\right)$

Hence, remainder $=P\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^3+3.\left(\frac{1}{2}\right)^2+3.\left(\frac{1}{2}\right)+1 =\frac{1}{8}+\frac{3}{4}+\frac{3}{2}+1$

$=\frac{1}{8}+\frac{9}{4}+1 =\frac{19}{8}+1=\frac{27}{8}$

The remainder when polynomial x^3+3x^2+3x+1 **is divided by** $x-\frac{1}{2}$ **is equal to** $\frac{27}{8}$

(iii) Given polynomial is $P(x)=x^3+3x^2+3x+1$

When $P(x)$ **is divided by** x**, then the remainder is** $P(0)$

Hence, remainder $=P(0)=(0)^3+3.(0)^2+3(0)+1 =1$

The remainder when polynomial x^3+3x^2+3x+1 **is divided by** x **is equal to** 1**.**

(iv) Given polynomial is $P(x)=x^3+3x^2+3x+1$

When $P(x)$ **is divided by** $x+\pi$**, then the remainder is** $P(-\pi)$

Hence, remainder $=P(-\pi)=(-\pi)^3+3.(-\pi)^2+3(-\pi)+1 = -\pi^3+3\pi^2-3\pi+1 = (-\pi+1)^3$

The remainder when polynomial $P(n)=x^3+3x^2+3x+1$ **is divided by** $x+\pi$ **is equal to** $(-\pi+1)^3$.

(v) Given polynomial is $P(x)=x^3+3x^2+3x+1$

When $P(x)$ **is divided by** $5+2x$**, then the remainder is** $P(-52)$

Hence, remainder = $P(-52)=(-52)^3+3.(-52)^2+3.(-52)+1 = -1258+3.(254)-152+1 = -1258+754-152+1$ $=258-152+1 = -358+1$

$=-278$

The remainder when x^3+3x^2+3x+1 **is divided by** $5+2x$ **is equal to** -278.

2. Find the remainder when x^3-ax^2+6x-a **is divided by** $x-a$.

Solution:

The remainder of polynomial $P(x)$ **when divided by another polynomial** $(ax+b)$ **where a and** b **are real numbers** $a\neq 0$ **is equal to** $P(-ba)$

Given polynomial is $P(x)=x^3-ax^2+6x-a$

When $P(x)$ **is divided by** $x-a$**, then the remainder is** $P(a)$

Hence, remainder = $P(a)=a^3-a.(a)^2+6(a)-a = a^3-a^3+6a-a = 5a$

The remainder when polynomial $P(x)=x^3-ax^2+6x-a$ **is divided by** $x-a$ **is equal to** $5a$

3. Check whether $7+3x$ **is factor of** $3x^3+7x$.

Solution:

Given polynomial is $P(x)=3x^3+7x$

For $7+3x$ **to be a factor of** $3x^3+7x$**, remainder when polynomial** $3x^3+7x$ **divided by** $7+3x$ **must be zero.**

We know, the remainder of polynomial $P(x)$ **when divided by another polynomial** $(ax+b)$**, where a and** b **are real numbers** $a\neq 0$ **is equal to** $P(-ba)$

Hence, remainder $=P(-73)=3(-73)^3+7.(-73)^2 = 3(34327)-493 = -3439-493 = -4909$

As remainder is not equal to zero

Hence $7+3x$ **is not a factor of** $3x^2+7x$