ST ANTONY'S CONVENT SCHOOL

Gagore Vijaypur

MATHEMATICS

CLASS : Xth

Q1 :

From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

(A) 7 cm (B) 12 cm

(C) 15 cm (D) 24.5 cm

Solution :

Let O be the centre of the circle.

Given that,

OQ = 25cm and PQ = 24 cm

As the radius is perpendicular to the tangent at the point of contact,

Therefore, OP ⊥ PQ

Applying Pythagoras theorem in ΔOPQ, we obtain

$OP^2 + PQ^2 = OQ^2$

$OP^2 + 24^2 = 25^2$

$OP^2 = 625 - 576$

$OP^2 = 49$

$OP = 7$

Therefore, the radius of the circle is 7 cm.

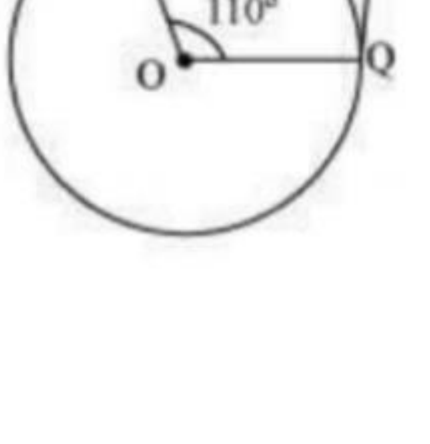
Hence, alternative (A) is correct.

Q2 :

In the given figure, if TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°, then ∠PTQ is equal to

(A) 60° (B) 70°

(C) 80° (D) 90°



Solution :

It is given that TP and TQ are tangents.

Therefore, radius drawn to these tangents will be perpendicular to the tangents.

Thus, $OP \perp TP$ and $OQ \perp TQ$

$\angle OPT = 90^\circ$

$\angle OQT = 90^\circ$

In quadrilateral POQT,

Sum of all interior angles = 360°

$\angle OPT + \angle POQ + \angle OQT + \angle PTQ = 360^\circ$

$\Rightarrow 90^\circ + 110^\circ + 90^\circ + \angle PTQ = 360^\circ$

$\Rightarrow \angle PTQ = 70^\circ$

Hence, alternative (B) is correct.

Q3 :

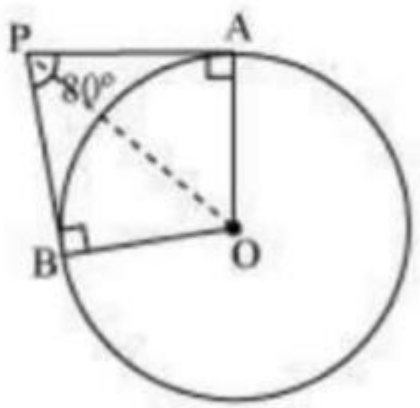
If tangents PA and PB from a point P to a circle with centre O are inclined to each other an angle of 80°, then ∠POA is equal to

(A) 50° (B) 60°

(C) 70° (D) 80°

Solution :

It is given that PA and PB are tangents.



Therefore, the radius drawn to these tangents will be perpendicular to the tangents.

Thus, $OA \perp PA$ and $OB \perp PB$

$\angle OBP = 90^\circ$

$\angle OAP = 90^\circ$

In AOBP,

Sum of all interior angles = 360°

$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^\circ$

$90^\circ + 80^\circ + 90^\circ + \angle BOA = 360^\circ$

$\angle BOA = 100^\circ$

In ΔOPB and ΔOPA,

AP = BP (Tangents from a point)

OA = OB (Radii of the circle)

OP = OP (Common side)

Therefore, $\Delta OPB \cong \Delta OPA$ (SSS congruence criterion)

A ↔ B, P ↔ P, O ↔ O

And thus, $\angle POB = \angle POA$

$$\angle POA = \frac{1}{2}\angle AOB = \frac{100^\circ}{2} = 50^\circ$$

Let AB be a diameter of the circle. Two tangents PQ and RS are drawn at points A and B respectively.

Radius drawn to these tangents will be perpendicular to the tangents.

Thus, $OA \perp RS$ and $OB \perp PQ$

$\angle OAR = 90^\circ$

$\angle OAS = 90^\circ$

$\angle OBP = 90^\circ$

$\angle OBQ = 90^\circ$

It can be observed that

$\angle OAR = \angle OBQ$ (Alternate interior angles)

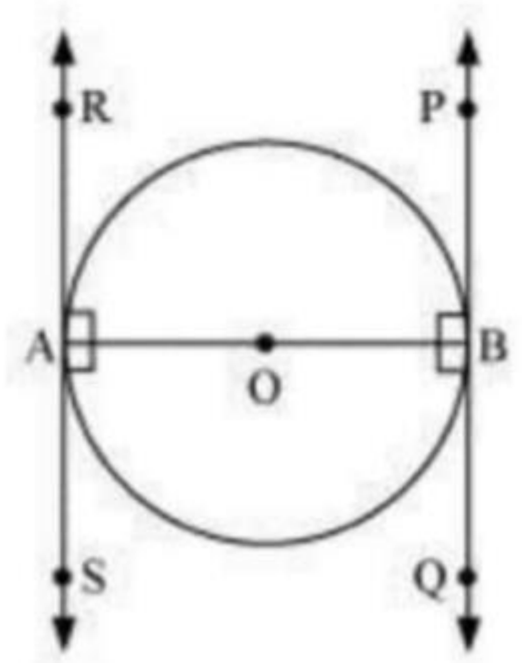
$\angle OAS = \angle OBP$ (Alternate interior angles)

Since alternate interior angles are equal, lines PQ and RS will be parallel.

Q4 :

Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Solution :

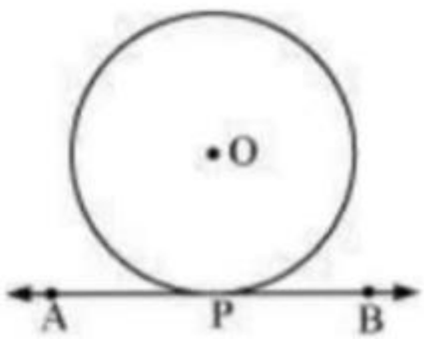


Q5 :

Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Solution :

Let us consider a circle with centre O. Let AB be a tangent which touches the circle at P.



We have to prove that the line perpendicular to AB at P passes through centre O. We shall prove this by contradiction method.

Let us assume that the perpendicular to AB at P does not pass through centre O. Let it pass through another point O'. Join OP and O'P.

As perpendicular to AB at P passes through O′, therefore,

$\angle O'PB = 90°$ ... (1)

O is the centre of the circle and P is the point of contact. We know the line joining the centre and the point of contact to the tangent of the circle are perpendicular to each other.

$\therefore \angle OPB = 90°$ ... (2)

Comparing equations (1) and (2), we obtain

$\angle O'PB = \angle OPB$ ... (3)

From the figure, it can be observed that,

$\angle O'PB < \angle OPB$ ... (4)

Therefore, $\angle O'PB = \angle OPB$ is not possible. It is only possible, when the line O′P coincides with OP.

Therefore, the perpendicular to AB through P passes through centre O.